FINAL DRAFT - MATH 20C - FALL 2020

Problem 1. (1 point each) Compute the following

(a) (1 point) $(1, 2, 1) \times (2, -3, 1)$.

(b) (1 point) $\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -5 \end{vmatrix}$

Problem 2.

(a) (2 points) What is the orthogonal projection of the vector $(9, 9)$ onto the vector $(2, 5)$?

(b) (3 points) Calculate the distance from the point $(1, 1, 1)$ to the plane given by the equation

$$2x + 4y - z = 0.$$

Problem 3. (4 points) Find an equation for the plane containing the points:

$$(1, 0, 1), (0, 2, 3), \text{ and } (0, 0, 2).$$

Problem 4.

(a) (2 points) Let

$$F(t) = (2t + 1, t, t^3 - 3t + 3)$$

Compute the derivative $DF(0)$. (Your answer should be a 3×1 matrix.)

(b) (2 points) Let

$$G(x, y, z) = x + e^{y+z}.$$

Compute the derivative $DG(1, 1, 2)$. (Your answer should be a 1×3 matrix.)

(c) (2 points) Compute the derivative:

$$D(G \circ F)(0)$$

using the chain rule. (Hint: Use your answers from part (a) and (b). Your answer should be a 1×1 matrix, or a scalar.)

Problem 5.

(a) (2 points) Find the critical points of

$$f(x, y) = 2x^3 - 3x^2 + y^2$$

(b) (3 points) Use the second derivative test to determine which of the critical points are local minimums, maximums, or saddle points.

Problem 6.

(a) (5 points) Use Lagrange multipliers to find the minimum and maximum values of

$$f(x, y) = 3x^2 - 2xy + 3y^2$$

on the set $x^2 + y^2 = 2$.

(b) (2 points) What are the global minimum and maximum values of $f(x, y)$ when $x^2 + y^2 \leq 2$?

Problem 7.

(a) (2 points) Sketch the region of integration in the integral

$$\int_{-1}^{0}\int_{2x}^{0} e^{2x-y}\,dy\,dx.$$

(b) (2 points) Change the order of integration for the integral in part (a), in other words, in other words write the integral in (a) as

$$\int_{c}^{d}\left(\int_{g_1(y)}^{g_2(y)} e^{2x-y}\,dx\right)dy.$$

for some constants c and d, and functions g_1 and g_2.

(c) (2 points) Evaluate the integral using either part (a) or part (b) (you do not need to solve twice).